ANALYSIS OF ELECTROTHERMAL TRANSIENTS AND DIGITAL SIGNAL PROCESSING IN ELECTRICALLY AND THERMALLY NONLINEAR MICROWAVE CIRCUITS

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ABSTRACT

The paper introduces a new approach to the analysis of nonlinear circuits containing temperature-dependent devices and excited by slowly modulated microwave carriers. For best accuracy and efficiency, the different dynamics of electrical and thermal phenomena are exploited. The electric circuit is simulated by a sequence of HB analyses of equal size, driven by an exact HB analysis loop for the thermal circuit.

INTRODUCTION

The steadily increasing importance of personal communication systems is stimulating the need for CAD techniques that can efficiently simulate nonlinear circuits and subsystems driven by digitally modulated RF/microwave carriers. In addition, the ability to evaluate the transient behavior of nonlinear circuits is also of considerable interest in a number of more traditional microwave engineering problems such as pulsed-RF operation of power circuits, phase acquisition in PLL's, oscillator and amplifier turnon, and so forth. From a CAD viewpoint, all these applications are special cases of a general simulation problem consisting in the analysis of nonlinear circuits supporting high-frequency quasi-periodic waveforms modulated by relatively low-frequency (possibly digital) signals. While a natural approach to this problem would be time-domain analysis, harmonic-balance (HB) with time-varying phasors has been independently proposed by several research groups [1] - [3] as a more efficient alternative. With these methods, microwave steady-state analysis is decoupled from envelope analysis, so that the well-known efficiency of HB techniques can be fully exploited. One important aspect of circuit operation that has not been taken into account until now in this class of simulations is the effect of device self-heating. Under modulated-RF drive, the active device temperatures are time-dependent, and may exhibit complex waveforms in relation with the RF regime, the shape of the signal envelopes, and the device thermal properties. In turn, this may have a major influence on the circuit performance, especially for power circuits. Very well-known examples are amplitude and phase droop in pulsed power amplifiers, but intermodulation as well may be affected by self-heating, even in simple multitone operation [4]. Thus for example it can be expected that thermal effects may give a contribution to the spectral regrowth and the adjacent-channel interference generated by nonlinear power amplifiers. In addition, the accurate evaluation of the time-dependent dissipated heat is by itself an information of primary importance for the design of some kind of communication equipment, such as portable transceivers. Thus the extension of ordinary HB with variable phasors [1] - [3] to cover electrothermal analysis is certainly worthwhile.

This task is accomplished here by an efficient frequency-domain technique based on a modified version of the modulation-oriented harmonic-balance (MHB) method introduced in [3]. Each active device is described by a nonlinear thermal equivalent circuit consisting of a resistance and a capacitance, and the heat sinking mechanism is simulated by a thermal transmission line [5]. The electrical state variables (SV) of the microwave circuit are described by truncated Fourier series with time-dependent phasors. The time-dependent device temperatures are described by ordinary Fourier expansions. The unknowns are determined by simultaneously solving two coupled nonlinear systems generated by the MHB technique [3] for the microwave circuit, and by the ordinary HB for the thermal circuit. Good numerical efficiency is achieved making use of a hierarchical solution algorithm. The capabilities of the new simulation technique are demonstrated by the electrothermal analysis of a nonlinear power amplifier driven by an 800 MHz carrier with $\pi/4$-DQPSK modulation.

THE ELECTROTHERMAL ANALYSIS ALGORITHM

Let us assume that the temperature-dependent nonlinear subnetwork consists of a collection of multiport semiconductor devices which are thermally isolated from each other, and that the electrical behavior of a generic device is essentially determined by one thermal state variable. This variable is expressed as $T_A + DT_D(t)$, where T_A is the known ambient temperature and $DT_D(t)$ is the excess temperature. The extension to the case of thermally coupled devices (or, equivalently, of devices requiring more than one thermal SV) is possible. The electric nonlinear subnetwork can be described by the set of parametric equations

$$\begin{aligned} \mathbf{v}(t) &= \mathbf{u}\left[\mathbf{x}(t),\ \frac{d\mathbf{x}}{dt},\ \mathbf{x_d}(t),\ T_A + \mathbf{DT_D}(t)\right] \\ \mathbf{i}(t) &= \mathbf{w}\left[\mathbf{x}(t),\ \frac{d\mathbf{x}}{dt},\ \mathbf{x_d}(t),\ T_A + \mathbf{DT_D}(t)\right] \end{aligned} \qquad (1)$$

where $\mathbf{v}(t)$, $\mathbf{i}(t)$ are vectors of voltages and currents at the device ports, $\mathbf{x}(t)$ is a vector of electrical SV, and $\mathbf{x_d}(t)$ is a vector of time-delayed electrical SV, i.e., $x_{di}(t) = x_i(t - \tau_i)$, τ_i being a time constant. $\mathbf{v}(t)$, $\mathbf{i}(t)$, $\mathbf{x}(t)$ have the same size n_D, equal to the total number of device ports. $\mathbf{DT_D}(t)$ is the vector of the device excess temperatures, and its size N_D equals the number of devices. (1) can easily accomodate advanced electrothermal models of microwave devices [6] and are thus sufficient for our present purposes; however, the generalization of the algorithm to more complex models including higher-order derivatives is straightforward [3]. The heat flow from each device active region to the ambient is modeled by a thermal transmission line (TL) of the kind shown in fig. 1. In this figure, D is the device node, B is the backplane metalization node, and A is the ambient node. Each R-C cell is representative of a section of the physical heat sinking structure,

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such as a soldering layer, a carrier, and the like. The TL can be analyzed by a dynamic electrical analogy based on the correspondence between heat flow (here represented by the symbol "q") and electric current, excess temperature and voltage. From fig. 1 we obtain the thermal equations of a generic device:

$$DT_B(t) = DT_D(t) + \gamma_D\,\theta_D[DT_D(t), DT_B(t)]\,\frac{dDT_D(t)}{dt} - \theta_D[DT_D(t), DT_B(t)]\,q(t)$$
$$q_B(t) = \gamma_D \frac{dDT_D(t)}{dt} - q(t) \qquad (2)$$

where γ_D, θ_D are the device thermal capacitance and thermal resistance, q(t) is the power dissipated in the device, and $DT_B(t)$ is the backplane temperature. The thermal resistance is temperature-dependent and can be explicitly formulated by means of the Kirchhoff transform [5]. The small dependence on temperature of the specific heat is practically negligible, so that the thermal capacitance is considered linear. Note that (2) have the same canonical form as (1), with thermal state variables $DT_D(t)$, $DT_B(t)$. The thermal nonlinear subnetwork is active, however, since it contains the state-dependent source q(t).

The electrical and thermal linear subnetworks may be described by frequency-domain equations of the form

$$\mathbf{I}(\omega) + \mathbf{Y}(\omega)\,\mathbf{V}(\omega) + \mathbf{Y_F}(\omega)\,\mathbf{F}(\omega) = \mathbf{0} \qquad (3)$$

$$\mathbf{Z_T}(\omega)\,\mathbf{Q_B}(\omega) + \mathbf{D_B}(\omega) = \mathbf{0} \qquad (4)$$

In (3) $\mathbf{Y}(\omega)$ is the admittance matrix at the device ports when all source ports are short-circuited, $\mathbf{Y_F}(\omega)$ is the forward transadmittance matrix from the source ports to the device ports, and $\mathbf{F}(\omega)$ is the vector of free sinusoidal voltage sources of angular frequency ω (*forcing terms*) connected to the source ports. Source and load resistances are included in the linear subnetwork. In (4) $\mathbf{Z_T}(\omega)$ is the diagonal matrix of the input thermal impedances of all TLs, and $\mathbf{Q_B}(\omega)$, $\mathbf{D_B}(\omega)$ are vectors of phasors of the spectral components of $q_B(t)$, $DT_B(t)$, for all devices at frequency ω. $\mathbf{Z_T}(\omega)$ is computed from fig. 1 by conventional linear circuit methods. The linear and nonlinear subnetwork equations (1) - (4) must be simultaneously solved (for all nonlinear devices) by the electrothermal analysis.

Let us now assume that the forcing terms are quasi-periodic microwave signals slowly modulated in amplitude and phase by baseband signals, i.e.,

$$\mathbf{f}(t) = \sum_{\mathbf{k}} \mathbf{F_k}(t)\,\exp(j\Omega_{\mathbf{k}} t) \qquad (5)$$

where $\Omega_\mathbf{k}$ is a generic intermodulation (IM) product of a set of RF/microwave fundamental frequencies ω_i, and the complex modulation laws $\mathbf{F_k}(t)$ are slowly varying with time. With the usual HB notation, **k** is a vector of integer harmonic numbers. The state vector **x**(t) has an expression similar to (5), with $\mathbf{F_k}(t)$ replaced by $\mathbf{X_k}(t)$. Since the thermal time constants of microwave semiconductor devices are normally large with respect to the RF period, the time-dependent device temperatures will only contain the baseband terms (**k** = **0**). Without loss of generality, we shall assume that all the $\mathbf{F_k}(t)$, and thus $\mathbf{X_k}(t)$, $\mathbf{DT_D}(t)$, $\mathbf{DT_B}(t)$ as well, are periodic or quasi-periodic. This allows the spectral properties of the modulation laws to be evaluated by the Discrete Fourier Transform (DFT).

Now let the modulation laws be sampled at a finite number of uniformly spaced time instants t_n ($1 \le n \le N$), that will be referred to as the modulation-law sampling (MS) instants. In order to find the electric nonlinear subnetwork response to **x**(t), we introduce a quasi-stationary approximation. We assume that the time constants of the microwave circuit are so short, that its electrical regime under modulated-RF drive can be described as a sequence of RF steady states [1] - [3], each associated with the set of values that the modulation laws and the device excess temperatures take on at a specific MS instant. Making use of (5) we get [3]

$$\frac{d\mathbf{x}(t)}{dt} = \sum_{\mathbf{k}} \left[j\Omega_\mathbf{k}\mathbf{X_k}(t) + \frac{d\mathbf{X_k}(t)}{dt} \right] \exp(j\Omega_\mathbf{k} t)$$
$$\mathbf{x_d}(t) \approx \sum_{\mathbf{k}} \exp(-j\Omega_\mathbf{k}\mathbf{T}) \left[\mathbf{X_k}(t) - \mathbf{T}\,\frac{d\mathbf{X_k}(t)}{dt} \right] \exp(j\Omega_\mathbf{k} t) \qquad (6)$$

where **T** is the diagonal matrix of the time delays τ_i. The computation of (6) requires the derivatives $d\mathbf{X_k}/dt$ which can be evaluated at the MS instant t_n by one-sided multipoint incremental rules of the general form

$$\left.\frac{d\mathbf{X_k}(t)}{dt}\right|_{t=t_n} \approx \sum_{m=0}^{M} a_m\,\mathbf{X_k}(t_{n-m}) \qquad (7)$$

where the coefficients a_m are explicitly listed in many mathematical handbooks. The accuracy of the derivatives, and thus of the MHB analysis, increases with M. However, it has been found that small values of M (say, $M \le 2$) are sufficient to obtain very satisfactory results in most practical cases. By means of (6), (7), and of the quasi-stationary approximation, the harmonics of (1) at each MS instant, namely $\mathbf{U_k}(t_n)$, $\mathbf{W_k}(t_n)$, can be computed by a multidimensional DFT as a function of $\mathbf{X_h}(t_{n-m})$ (for all **h** and $0 \le m \le M$) [3].

Under the same assumptions, if Ω is an angular frequency falling inside the band of the modulation laws, a frequency change by Ω will produce a very small modification of the microwave circuit admittance. Thus in the neighborhood of each $\Omega_\mathbf{k}$ the admittance matrices $\mathbf{Y}(\omega)$, $\mathbf{Y_F}(\omega)$ may be approximated by truncated Taylor expansions. By combining such expansions with the linear subnetwork equations and replacing the harmonics $\mathbf{U_k}(t_n)$, $\mathbf{W_k}(t_n)$ of (1) for $\mathbf{I}(\omega)$, $\mathbf{V}(\omega)$ in (3), we obtain a nonlinear system for the SV harmonics at a generic MS instant t_n. The mathematical developments are similar to those reported in [3], and will not be repeated here. An important difference, however, is that here we make use of the one-sided formula (7) for the derivatives, instead of the two-sided formula used in [3]. Each MS instant is then coupled to a finite number of other MS instants that precede it in time. If the real and imaginary parts of the HB errors (for all **k**) are stacked into a real electrical error vector $\mathbf{E}_n$, the electrical HB system for $t = t_n$ may then be symbolically stated in the form

$$\mathbf{E}_n[\mathbf{Z}_n, \mathbf{Z}_{n-1}, \ldots, \mathbf{Z}_{n-2M}, \{\textit{thermal unknowns}\}] = \mathbf{0} \qquad (8)$$

where $\mathbf{Z}_n$ is the vector of real and imaginary parts of the $\mathbf{X_k}(t_n)$ (for a fixed n and all **k**). The entries of $\mathbf{Z}_n$ (for all n) represent the electrical unknowns. The set of all electrical unknowns will be denoted by **Z**. Note that the nonlinear operator $\mathbf{E}_n[\bullet]$ is obviously temperature-dependent, as well, so that the system

(8) also contains the thermal unknowns. However, the latter are different in nature from the electrical unknowns, as explained by the following discussion.

The quasi-stationary assumption leading to (8) cannot be used in the nonlinear analysis of the thermal circuit. This is due to the fact that the thermal circuit time constants are rather long, ranging from several μs (for the active devices) to several ms (for a real heat sink), and thus typically have the same order of magnitude as the time constants of the modulation laws. For instance, in the NADC system [7] the RF carrier is modulated at a rate of 49 kb/s, so that the bit interval is about 20.4 μs. Thus the thermal circuit electrical regime is fully dynamic, and must be analyzed by a rigorous HB approach. Since by assumption the modulation laws are quasi-periodic, they can be expressed by multiple Fourier expansions of the form

$$\mathbf{F}_k(t) = \sum_s \mathbf{F}_{s,k} \exp(j\Omega_s' t) \qquad (9)$$

where the Ω_s' are IM products of a set of baseband fundamental frequencies ω_i'. All time-dependent thermal quantities may be represented by similar expansions. In particular, for the N_D-vectors of the peak and backplane device excess temperatures, $\mathbf{DT_D}(t)$, $\mathbf{DT_B}(t)$, the s-th harmonics will be denoted by $\mathbf{D_{Ds}}$, $\mathbf{D_{Bs}}$. The vector containing the real and imaginary parts of all harmonics $\mathbf{D_{Ds}}$, $\mathbf{D_{Bs}}$, will be denoted by $\mathbf{D}$. The entries of $\mathbf{D}$ are the thermal unknowns.

At the MS sampling instant t_n $(1 \le n \le N)$, the power dissipated in the r-th device $(1 \le r \le N_D)$ is given by

$$q_r(t_n) = \mathrm{Re}\left[\mathbf{U_0}^T(t_n)\,\mathbf{W_0}(t_n) + \frac{1}{2}\sum_{k\neq 0} \mathbf{U_k}^T(t_n)\,\mathbf{W_k}^*(t_n)\right]^{(r)} \qquad (10)$$

where the superscript (r) indicates that the vector products are only extended to the voltage and current harmonics at the r-th device ports. A Fourier expansion of the form (9) for $q_r(t)$ may then be computed by the DFT. The formulation of the thermal HB equations is now straightforward, and follows the same guidelines as for a conventional HB analysis [8]. The real and imaginary parts of the thermal HB errors are stacked into a real thermal error vector $\mathbf{E_T}(\mathbf{Z}, \mathbf{D})$, so that the electrothermal MHB solving system may be written in the form

$$\begin{cases} \mathbf{E}_n[\mathbf{Z}_n, \mathbf{Z}_{n-1}, \ldots, \mathbf{Z}_{n-2M}, \mathbf{D}] = \mathbf{0} \\ (1 \le n \le N) \\ \\ \mathbf{E_T}(\mathbf{Z}, \mathbf{D}) = \mathbf{0} \end{cases} \qquad (11)$$

(11) is a system of $N_U = Nn_D(2n_H + 1) + 2N_D(2n_T + 1)$ real equations in as many real unknowns, where n_H, n_T are the numbers of IM products used to describe the microwave steady-state regime and the time-dependent thermal quantities.

SOLUTION OF THE NONLINEAR SYSTEM

In many practical cases (such as for digitally modulated carriers) the number of MS instants may be quite large, so that the number of unknowns may climb up to several tens of thousands or more [1] - [3]. Thus in order to avoid exceedingly large CPU times, a clever solution strategy for the system (11) is needed. It turns out that this system is ideally suited for the application of the hierarchical technique introduced in [9]. n_T is normally less than N/2, but can often be further reduced if the fine details of the temperature waveforms are not of specific interest. In addition, $N_D \le n_D$. Thus the size of the thermal subsystem in (11) is much smaller than N_U. Following [9], we may assign to the thermal subsystem the role of master system, or equivalently solve the nonlinear system

$$\mathbf{E_T}[\mathbf{Z}(\mathbf{D}), \mathbf{D}] = \mathbf{0} \qquad (12)$$

where $\mathbf{Z}(\mathbf{D})$ is the solution of the electrical subsystem for a given $\mathbf{D}$. In turn, the electrical subsystem can be efficiently solved thanks to its very peculiar structure. As a matter of fact, for a given set of thermal unknowns, (8) can be viewed as a real system of $n_D(2n_H + 1)$ equations in as many unknowns (the entries of $\mathbf{Z}_n$), with $\mathbf{Z}_{n-1}, \ldots, \mathbf{Z}_{n-2M}$ playing the role of parameters. Thus with a suitable intialization, the electrical subsystem can be solved as a sequence of N independent ordinary HB systems of size $n_D(2n_H + 1)$.

In practice, (12) is solved by a Newton iteration starting from $\mathbf{D} = \mathbf{0}$. For a given $\mathbf{D}$ the device excess temperatures can be evaluated at all the MS instants. The electrical subsystem may then be solved starting from the solutions obtained at the previous iteration ($\mathbf{Z} = \mathbf{0}$ at startup). At the first MS instant t_1, an ordinary HB analysis is carried out. At the MS instants t_m $(2 \le m \le 2M)$ the expressions (7) of the derivatives are suitably simplified (i.e., by reducing M), so that the computation only requires already available information from the preceding MS instants. For $n > 2M$ the standard system (8) is sequentially solved for increasing values of n. At the end of this process, the heat sources are evaluated by (10) (for all r, n) and Fourier transformed, which provides the forcing terms of the thermal HB equations. The thermal HB errors may then be computed. The Jacobian matrix of (12) is exactly evaluated by algorithms similar to those discussed in [8], [9]. Since (12) is only mildly nonlinear, 2-3 iterations with respect to the thermal unknowns are usually sufficient to achieve convergence, so that the overall solution process is numerically efficient.

AN EXAMPLE OF APPLICATION

As an example of application, we consider the electrothermal analysis of a power amplifier driven by a digitally modulated carrier. The carrier frequency is 835 MHz and the modulation format is π/4-DQPSK according to the NADC standard [7]. The amplifier consists of a simple class-A FET stage operated at 4 dB gain compression, and the active device is modeled after [6]. The device is assumed to be mounted in a microstrip package, whose ground plane is soldered to a metal heat sink of finite size. Three layers representing the package, the solder, and the finite heat sink, are thus included in the thermal TL between the device backplane and the ambient. The thermal conductivity of GaAs is assumed to be proportional to $T^{-1.2}$ [5]. The baseband bit stream is described as a periodic sequence of 512 bits with a bit rate of 49 kb/s. Figs. 2, 3 show the input and output normalized power spectra of the modulated carrier, for an input power of +23.25 dBm on the useful channel. The output spectrum is shifted by an amount equal to the amplifier gain for ease of comparison. The spectral regrowth of the output signal due to IM distortion in the power amplifier is evident from these figures. The power gain and power-added efficiency of the amplifier are obviously influenced by the device self-heating. With digital modulation and +23.25 dBm input power on the useful channel, the gain and efficiency values with the device at ambient temperature (290 °K) are G = 7.4 dB, η = 34.4 %. On the other hand, when self-heating is accounted for the gain drops to G = 6.1 dB, with a power-added efficiency η = 27.7 %. Fig. 4 shows the in-phase and quadrature components of the output modulation laws and the FET temperature waveform $T_A + DT_D(t)$ in a 64-bit slot extracted from the main sequence. The FET thermal time constant is about 120 µs at 290 °K. The analysis makes use of 4 sampling points per bit and 4 carrier harmonics plus d.c., so that N = 2048, and the number of electrical unknowns is 36864. $n_T = 512$ thermal harmonics are used, so that the number of thermal unknowns is 2050, and $N_U = 38914$. The CPU time is 1136 seconds on an HP 755 workstation.

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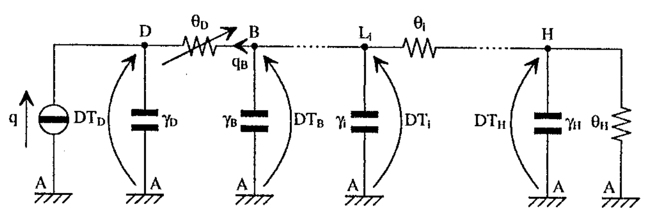

Fig. 1 - Thermal transmission-line model of the heat flow from an active device.

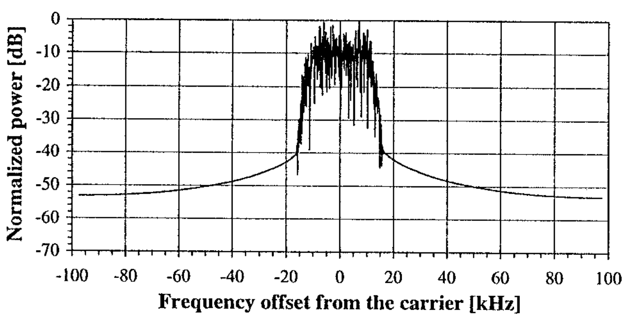

Fig. 2 - Input power spectrum.

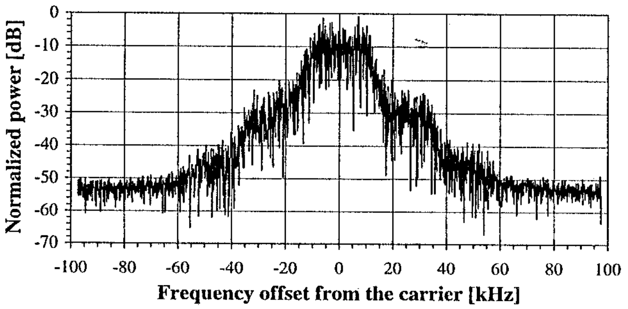

Fig. 3 - Output power spectrum.

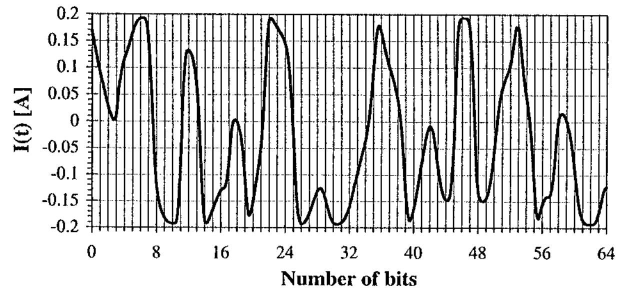

Fig. 4a - In phase modulation law of the load current.

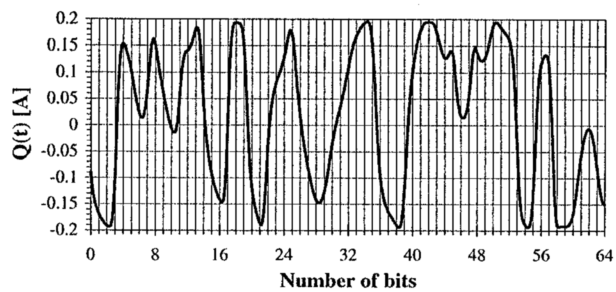

Fig. 4b - Quadrature modulation law of the load current.

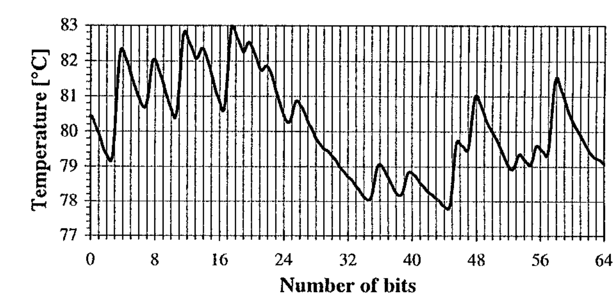

Fig. 4c - Peak device temperature.